How Limiting Deceptive Practices Harms Consumers

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1 Equilibrium and welfare for high quality differentials.

By imposing assumption **A1** we have restricted the analysis to the range of parameters in which $v_l > \frac{\Delta}{2}$. This section provides the equilibrium analysis for the opposite case in which the differential between the sellers' qualities is not small. Section 1.1 deals with the basic model while Section 1.2 assumes that the buyer is more sophisticated as in Section 5 of the paper.

1.1 Basic model

Take the baseline model of the paper and assume that **A1** does not hold. Consider first the separating equilibrium of Proposition 1. The arguments which pin down prices $p_h = \Delta$ and $p_l = 0$ are still valid. What needs to be checked anew is the no-mimicking condition. Start observing that, if f is larger than Δ, mimicking cannot be profitable because it implies a penalty larger than the price if the buyer purchases. Thus, in the region in which $f \geq \Delta$ the separating equilibrium exists even if **A1** does not hold. Focus next on the area in which the sanction f is smaller than the separating price Δ. Notice that, if $v_l \leq \frac{\Delta}{2}$ and the low-quality seller mimics the high-quality seller charging a price Δ, the buyer expects quality $v_l + \frac{\Delta}{2} \leq \Delta$ and abstains from purchasing. Thus, the separating equilibrium exists in this region.[1] Summing up, the separating equilibrium of Proposition 1 exists for any level of the sanction f as long as $v_l \leq \frac{\Delta}{2}$.

Consider now pooling equilibria. The buyer's off-equilibrium beliefs are still those of **A3**. Notice first that nothing changes on the sellers' side: their no-deviation incentives are the same as those of the main text and do not depend on v_l. Hence, sellers do not deviate as long as the participation constraint and the no-deviation constraint of the low-quality seller are satisfied — i.e., (3) and (4) are met. The substantial difference with respect to the analysis of the main text rests with the buyer participation constraint. In particular, it is no longer true that the buyer's participation constraint (2) is satisfied at the minimum price f whenever the low-quality seller's participation and no-deviation constraints, (3) and (4), are met. As a consequence, the sellers' incentives might be satisfied for prices which violate the buyer's participation constraint. This simply means that, when **A1** does not hold, the buyer's participation constraint (2) must be explicitly accounted for. While, as before, (3) and (4) are jointly

[1] For simplicity, we assume the buyer does not purchase in the non generic case $v_l = \frac{\Delta}{2}$ — e.g., because there is a minimal cost of shopping. Assuming that he buys would not change the main welfare and policy implications of the paper.

satisfied only if $f \leq \Delta$, (2) and (3) are jointly satisfied only if $f \leq v_l + \frac{\Delta}{2}$. The latter is a new constraint imposed on the model's parameters for the existence of a pooling equilibrium and is clearly stricter than the former. It reflects the fact that, because **A1** does not hold, the buyer's participation constraint is tighter.

Summing up, it is immediate to verify that the following result holds:

Proposition I. *If assumption* **A1** *does not hold:* (i) *assuming* **A2** *the separating equilibrium of Proposition 1 exists for any* f*;* (ii) *assuming* **A3** *the most competitive pooling equilibrium with price* f *exists if and only if* $f \leq v_l + \frac{\Delta}{2}$.

Beyond the appearance of the participation constraint in the existence condition of the pooling equilibrium, the main addition of Proposition I to the analysis of Section 2.1 regards the separating outcome. If **A1** is relaxed the separating equilibrium of Proposition 1 extends its range of application. Indeed, when the quality differential is sufficiently large — i.e., $v_l \leq \frac{\Delta}{2}$ — this equilibrium exists for any enforcement level. The intuition is simple. In this region of parameters, if the low-quality seller mimics the high-quality one and both advertise, the quality expected by the buyer under **A2** ($v_l + \frac{\Delta}{2}$) is lower than the price of the high-quality good at the separating equilibrium (Δ). Hence, the buyer does not purchase off-equilibrium and the low-quality seller has no incentives to deviate.

A comment on welfare is due. When multiple equilibria exist, we bear with the selection criterion that sellers most likely coordinate on the one which maximizes their *ex-ante* profit — i.e., before learning their type. When $v_l \leq \frac{\Delta}{2}$, the pooling and the separating equilibria coexist for $f \leq v_l + \frac{\Delta}{2}$ while only the separating exists for larger levels of the sanction. It is then immediate to verify that a seller's expected profits are, respectively, $\frac{f}{2}$ in the pooling equilibrium and $\frac{\Delta}{2}$ in the separating one. Hence, the latter dominates the former and sellers coordinate on the separating equilibrium for any value of f. In this scenario, the conflict of interest between the buyer and the sellers is the most severe: while the buyer would like sellers to coordinate on the most competitive pooling equilibrium with zero enforcement, sellers are never willing to do so. Clearly, because sellers coordinate on the separating outcome no matter f, the Authority cannot affect the buyer-surplus, which is stuck at v_l. Indeed, the Authority could benefit the buyer only by inducing sellers to coordinate on a pooling equilibrium and then setting the fine at the minimum level $f = 0$ — replicating, in fact, the result of Proposition 3. To do so the Authority should implement a price-cap $\overline{p}$ sufficiently low to induce a low-quality seller to deviate from her separating price of zero and mimic the high-quality seller, thereby destroying the separating equilibrium. It is easy to see that $\overline{p}$ will be optimally set at $v_l + \frac{\Delta}{2}$ — i.e., the buyer's maximum willingness to pay off-equilibrium implied by assumption **A3**. Then, the policy of the buyer-oriented Authority requires banning any price $p \geq \overline{p}$ and setting a fine $f = 0$.

1.2 Sophisticated buyer

We now assume that the buyer is more sophisticated and realizes that a high-quality seller can credibly signal her type by pricing below f *and* advertising.

Note first that the analysis of the separating equilibrium of Section 1.1 holds unchanged when the buyer is more sophisticated. Hence, point (*ii*) of Proposition I applies here as well. This is relevant for the welfare analysis at the end of the subsection.

Second, consider pooling equilibria. The analysis is almost identical to that developed in the main text of the paper. The buyer's off-equilibrium beliefs are still those of **A4**. The incentives to be satisfied are, again, the buyer's participation constraint (2), the no-self-revealing condition of the high-quality seller (5) and the no-deviation constraint of the low-quality seller (4).[2] Again, (4) and (5) are jointly satisfied only if $f \leq \frac{2}{3}\Delta$, while (2) and (5) are jointly met only if $f \leq \frac{1}{2}\left(v_l + \frac{\Delta}{2}\right)$. Notice that, because **A1** does not hold and $v_l \leq \frac{\Delta}{2}$, the latter condition is stricter than the former. This is the only difference with respect to Proposition 7, in which there is no straight implication between the necessary conditions just mentioned. Hence, a necessary and sufficient condition for the existence of a pooling equilibrium with sophisticated buyer and minimum price $p^* = 2f$ is:

$$f \leq \hat{f} \equiv \frac{1}{2}\left(v_l + \frac{\Delta}{2}\right).$$

Finally, consider the separating equilibrium of Proposition 8. Notice first that, assuming **A5** and considering an equilibrium in which sellers price at f and separate on the advertising strategy, the incentives to deviate of either seller are unaffected by the relationship between v_l and Δ. In particular,

[2] The participation constraint of the low-quality seller (3) is implied by (5) and the no-deviation constraint of the high-quality seller (6) is implied by (4).

this is so because, independently of v_l, the low-quality seller earns zero profits by mimicking the rival. As to the buyer, he is clearly willing to purchase the high-quality good at price $f < \Delta$ in equilibrium, which is the (unchanged) existence condition of this separating outcome. Hence, the separating equilibrium of Proposition 8 is unaffected by dropping **A1**.

We summarize the analysis of Section 1.2 up to this point in the following result.

Proposition II. *If assumption* **A1** *does not hold, then: (i) assuming* **A4** *the most competitive pooling equilibrium with price* $p^* = 2f$ *exists if and only if* $f \leq \hat{f}$*; (ii) assuming* **A5** *the separating equilibrium of Proposition 8 exists under the same condition* $f \leq \Delta$*.*

Hence, the only change *vis-à -vis* the analysis of Section 5 is the simplified existence condition of the pooling equilibrium.

The welfare analysis closely parallels that of Section 5. The only difference is that now the separating equilibrium with price $p_h = \Delta$ exists for any f. The consequence of this is that the separating equilibrium with price $p_h = f$ is never played by the sellers, who obviously prefer playing the separating equilibrium with the higher price. Hence, whenever they coexist — i.e., if $f < \hat{f}$ — the relevant comparison is between the pooling outcome and the separating with price Δ. In the former a seller expects profits equal to $\frac{3}{4}f$ while she expects profits equal to $\frac{\Delta}{2}$ in the latter. Notice that $\frac{\Delta}{2} \geq \frac{3}{4}f$ if and only if $f \leq \frac{2}{3}\Delta$, which is always true whenever the pooling equilibrium of Proposition II exists and **A1** does not hold. Hence, sellers always coordinate on the separating when they have a choice. Therefore, the same logic of Section 1.1 applies here and a price cap $\overline{p}$ capable of disrupting the separating equilibrium with price Δ should be implemented. As a consequence, sellers are left with a choice between the separating at price f and the pooling equilibrium at price $2f$. Hence, the analysis developed for the case $v_{>}\frac{\Delta}{2}$ applies. Summarizing, the optimal policy of a buyer-concerned Authority when the buyer is sophisticated is to set $f = 0$ and, if $v_l < \frac{\Delta}{2}$ a price cap $\overline{p} = v_l + \frac{\Delta}{2}$.