Cheap Talk with Correlated Information

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Abstract

We consider a situation where a decision maker gathers information from two or more imperfectly informed experts. Private information is (conditionally) correlated across players, and communication is cheap talk. We show that with two experts correlation unambiguously tightens the conditions on preferences for a truth-telling equilibrium. However, with multiple experts the effect of correlation on the incentives to report information truthfully is non-monotonic. In particular, while very small and very large correlation levels are detrimental for truth-telling, intermediate levels of correlation may *discipline* experts' equilibrium behavior, making it easier to sustain truth-telling.

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1 Introduction

The importance of strategic information transmission in economic contexts has long been recognized. Following the seminal work of Crawford and Sobel (1982), a large body of literature has studied the conditions under which information is transmitted truthfully in equilibrium, and the impact of different informational and strategic conditions. Much of the theoretical effort has focused on possibility to sustain fully-revealing equilibria, in which the informed agents truthfully report the information they possess. An ubiquitous assumption in this literature is that, when agents are partially informed, their information is conditionally independent. In this paper we add to the debate by reconsidering this crucial assumption, and study the role of signals' correlation in cheap talk games. Our main contribution is to show that the effect of signals' correlation on the incentives to reveal information is non-monotonic. While both too little and too much correlation impair information transmission, some amount of correlation may facilitate it. This potential role of correlation to facilitate communication in strategic environments has not been explored to date.[1]

Correlation is, indeed, a common feature of many problems in which information is transmitted strategically: experts audited by a legislative commission may have partially coincident information sources; in discussion networks, people tend to take similar political stands as their neighbors, and to form their opinions sourcing from similar media; experts consulting a governmental agency on a specific economic issue may report on evidence obtained, at least in part, from the same reports and bulletins; the information of witnesses in a cross-examination may overlap to different extents; etc. In general, correlation of private information may be caused either by external factors (e.g. a small number of available information sources) or by agents' choices (e.g. people with similar views, preferences or backgrounds may draw information

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[1]Battaglini (2004) writes: "whether the assumption of conditionally independent signals can be relaxed, and to what extent [one can obtain] information aggregation even when residual errors are correlated are open questions that we leave for future research" (Remark 4, p. 16.).

from similar sources and have similar acquaintances). While it is clear that correlation weakens the welfare gains from information transmission, the way it affects the incentives to strategically disclose truthful information is not obvious *a priori*.

We address this problem in the context of a cheap talk game where an uninformed decision maker (she, also referred to as receiver) simultaneously gathers information from two or more partially informed experts (or senders) before taking action. We model strategic communication adopting the binary signals framework introduced by Austen-Smith (1990) in the context of information aggregation. In that paper, the signals of the informed parties are conditionally independent. We generalize the analysis to account for conditionally correlated signals. We first study a model with a decision maker and two experts. We show that correlation unambiguously tightens the existence conditions for a truth-telling equilibrium. The intuition is straightforward: by decreasing the informational content of each signal, correlation weakens the impact of information transmission on the receiver's decision and minimizes the risk of an excessive reaction — the so called overshooting effect, see Morgan and Stocken (2008). This, in turn, increases an expert's temptation to influence the decision maker's action, and this greater temptation makes more biased experts less credible. This result is general: we show, using results from Bahadur (1961), that the distribution we use is the unique symmetric joint distribution of two binary signals conditional on the state of the world. This distribution can be represented as the outcome of an information generating process in which both experts observe perfectly correlated signals (or, equivalently, the same signal) with some probability, and i.i.d. signals with complementary probability. As it turns out, the correlation between signals equals the probability that players observe perfectly correlated information in the process just described.

We then generalize the information structure to the case of more than two experts, and characterize the incentive constraints defining a truth-telling equilibrium. We note that the effect of an off-equilibrium message (a lie) on the decision maker's

action crucially depends on the correlation structure of the joint distribution of signals and on the number of experts. This implies that truth-telling constraints may differ substantially from the standard case of independence or even from the case of correlation with only two informed experts mentioned above.

To substantiate our claim, we investigate these constraints in the context of the (conditional) joint distribution of signals that extends to n experts the information generating process described for the case with two experts. Once again, signals either come from perfectly correlated sources with some probability or, with complementary probability, they come from a binomial distribution. This process generates a joint distribution which features equal marginals and constant pairwise correlation among any two signals, as in the case with two experts. We find that the effect of correlation on the incentives to reveal information is non-monotonic. For low correlation values, an increase in correlation tightens the requirements that truth-telling imposes on preferences. After a critical level of correlation is reached, however, further increases relax the required similarity in preferences, until a specific correlation level is reached, at which a truth-telling equilibrium exists irrespective of preferences. If we let correlation grow even larger than this threshold level, the requirements for truth-telling tighten again, until the trivial case of perfect correlation is reached.

This non-monotonic pattern is driven by two conflicting forces that shape an expert's incentives to misreport his signal. The first is the *direct* effect that one additional piece of information has on the decision maker's action: a "high" ("low") signal increases (decreases) the action. The second is an *indirect* effect, and depends on the way the receiver interprets signals. Observing mixed reports induces the receiver to believe that signals come from independent sources; this belief makes the receiver more reactive to all signals than in the case of aligned reports (consistent, therefore, with both i.i.d. and perfectly correlated processes). This indirect effect potentially may drag the receiver's action down in response to the report of a "high" signal when all other experts report "low" signals, as a consequence of the more intense reaction

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to all reports. A similar logic applies to the reaction to a "low" report added to a series of "high" reports. Key to our results is the fact that both the odds and the magnitude of this indirect effect increase with the level of signals' correlation, relaxing the constraints for truth-telling when correlation is below a threshold level, and tightening such constraints above this threshold. Hence the non-monotonic pattern discussed above.

We also show that the indirect effect becomes prevalent when the number of experts grows, and that when this number grows without bound, the maximal bias in preferences still consistent with truth-telling is bounded away from zero. This is a direct consequence of signal's correlation, and is in stark contrast with the traditional impossibility result for the case in which the number of experts grows large (see Morgan and Stocken (2008) and Galeotti, Ghiglino, and Squintani (2013)).

The paper is organized as follows: next we discuss the related literature. Section 2 introduces the model with two experts and characterizes the truth-telling equilibrium with correlated information in the simplest environment. It also presents the uniqueness result on the joint distribution of two correlated binary signals. Section 3 generalizes the model to more than two experts: we first characterize truth-telling for general (symmetric) distributions, and then focus on a specific distribution for which we explicitly derive the incentive constraints for a truth-telling equilibrium as a function of the level of correlation. Section 4 concludes.

1.1 Related Literature

Throughout the years a large literature has built on the seminal paper by Crawford and Sobel (1982) to study strategic information transmission by informed agents when communication is cheap talk. In what follows we briefly review a small subset of the most relevant literature.

A first set of papers assumes that the state of the world is perfectly known to the senders. Under the assumption of perfect information several forces through which

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truth-telling can be achieved have been identified. Krishna and Morgan (2001b) build on Gilligan and Krehbiel (1989) and show that a fully-revealing equilibrium can be achieved in a cheap talk model with multiple referrals and asymmetric information on a single dimension. Both papers study information transmission in the context of legislation under different settings: "closed rule", whereby the decision maker takes an action between the ones proposed or no action, and "open rule", in which the decision maker's action is unconstrained. Krishna and Morgan (2001a) extend the analysis to sequential elicitation of information by two experts and find that full revelation of information can be achieved when experts are allowed to argue and counter-argue through rebuttal. Battaglini (2002) derives an important result: he shows that full revelation is generically feasible in a model with two experts when information is elicited along multiple dimensions. The result hods no matter the size of the biases provided that they are not in the same direction. To fully extract information the receiver exploits the heterogeneity in senders' preferences on the dimension each one cares the most and weights each sender's report accordingly. Ambrus and Takahashi (2008) show that the hypothesis that the state space is unbounded is critical to Battaglini (2002)'s result: when the state space is restricted and biases are relatively large, there is no fully-revealing equilibrium.

Beginning with Austen-Smith (1990), a different strand of the literature focuses on the case in which the experts are only imperfectly informed about the state. This assumption, which we also make, is plausible in many circumstances. Austen-Smith (1990) studies debate and agenda setting in a sequential framework, while Austen-Smith (1993) compares the informative properties of multiple referrals under open rule when referrals are simultaneous or sequential. In neither paper full revelation of the state is attained when multiple experts are consulted and Austen-Smith (1993) finds that more information is extracted via sequential as opposed to simultaneous communication. Battaglini (2004) extends the analysis to an arbitrary number of experts holding continuous signals on a multidimensional state. He finds that a ten-

sion emerges between information aggregation and information extraction: a decision maker may reduce noise by aggregating a larger number of signals, but at the cost of receiving less truthful signals from each expert. Gerardi, McLean, and Postlewaite (2009) consider aggregation of experts' opinions when their preferences are private information. They develop a mechanism by which the decision maker achieves almost full information extraction when experts' signals are accurate and the number of experts is high. To achieve the result the decision maker commits to slightly distort his optimal action. Taking a different perspective, Ottaviani and Sørensen (2006a,b,c) study professional advice when, after the messages have been sent, the state is revealed to the receiver. In these models, the experts care about the reputation on their ability rather than the action taken by the decision maker. Hence, because the accuracy of signals grows with ability and the difference between their message and the state is eventually learnt, experts refrain from truth-telling to avoid too harsh inference on ability. We further notice that Ottaviani and Sørensen (2006b) has explicitly dealt with the impact of the *quality* of information on truth-telling behavior when information is transmitted between two players. The effect of the quality of information, measured by its accuracy, plays a role which is somewhat similar to that of correlation, by determining the magnitude of the receiver's reaction.

Our set-up is also somewhat related to the strand of literature on auctions and mechanism design focusing on the effect of correlated information (affiliated values) on the ability of the principal to extract all available information (and surplus) from the agents (see Myerson (1981) and Crémer and McLean (1985)). McAfee and Reny (1992) has extended this result to general mechanisms beyond auctions. The idea behind these findings is simple: the auctioneer offers a fee schedule detailing the amount to be paid by a bidder conditional on his and, crucially, on other bidders' reports. The fee schedule is such that a bidder expects to pay the least when he reports his true value. The auctioneer can give the right incentives because, due to affiliation, the conditional distribution of other bidders' values depends on a bidder's private

value. Our contribution is framed in the context of information transmission games, and therefore not directly comparable with the case in which a principal has full commitment on the information extraction mechanism and can operate transfers. It is, however, a remarkable similarity how the correlation of experts' private information can *discipline* their behavior even in a context where the commitment embodied in a mechanism is missing.

2 Two Informed Experts

We begin our analysis with the simplest benchmark in which an uninformed receiver collects information from two informed senders. We later move to the case with n senders. We use the framework with imperfect and coarse information introduced by Austen-Smith (1990) and widely adopted in the literature.

Players. We denote by R the decision maker (she) and by $i = 1, 2$ each expert (he). The receiver takes an action $y \in \mathbb{R}$ that affects the utilities of all players. These also depend on the state of the world θ, which is unknown to all players. Before the receiver takes action, each sender observes a binary signal about the state θ. We denote the senders' signals as $s_i \in \{0, 1\}$. After observing the signal, each sender i privately sends a message $t_i \in \{0, 1\}$ to the receiver. After hearing all messages, the receiver takes action y. We consider quadratic loss utility functions: the utility of the receiver is $U^R(y, \theta, b_R) = -(y - \theta - b_R)^2$ and the utility of sender i is $U^i(y, \theta, b_i) = -(y - \theta - b_i)^2$, with b_R and b_i representing individual preferences.

Information structure. The state of the world is known to be uniformly distributed on the unit interval, $\theta \sim U[0, 1]$. Conditional on the state, senders' signals are drawn from a joint distribution $\Pr(s_1, s_2 | \theta)$. Both senders have equally informative signals such that $\Pr(s_i = 1 | \theta) = \theta$, $i = 1, 2$. Using the representation theorem of Bahadur (1961), the following lemma fully characterizes the unique joint distribution as a function of the state and the pairwise correlation parameter k.

Lemma 1. *Let* $\Pr(s_i = 1|\theta) = \theta$, $i = 1, 2$ *and* $k = \frac{Cov(s_1, s_2)}{\sigma_{s_1}\sigma_{s_2}}$, *then the joint distribution of* s_1 *and* s_2 *given the state* θ, $\Pr(s_1, s_2|\theta)$, *is unique and is reported in Table 1.*

Table 1: Signals joint distribution conditional on θ

	$s_2 = 0$	$s_2 = 1$
$s_1 = 0$	$(1-\theta)k + (1-\theta)^2(1-k)$	$\theta(1-\theta)(1-k)$
$s_1 = 1$	$\theta(1-\theta)(1-k)$	$\theta k + \theta^2(1-k)$

It is important to notice that the joint conditional probability distribution just introduced is well defined only for non-negative values of the correlation parameter — i.e., it must be $k \in [0, 1)$. To see why, notice that the NW and SE quadrants of Table 1 are non-negative for all $\theta \in [0, 1]$ if and only if $k \geq 0$.

The joint distribution characterized in Lemma 1 can be viewed as originating from the following compound lottery: either a draw from a binomial distribution $B(2, \theta)$ is selected with probability $1 - k$, or, with probability k, a draw is selected from a degenerate joint distribution corresponding to a single Bernoulli experiment with success probability equal to θ. Put differently, the process generating $\Pr(s_1, s_2|\theta)$ is one in which senders either collect information from independent sources with probability $1 - k$ or, with probability k, they collect information from perfectly correlated sources — or, equivalently, the same source. All information sources are such that $\Pr(s_i = 1|\theta) = \theta$, $i = 1, 2$.

Finally, since θ is uniformly distributed the marginals are $\Pr(s_i = 1) = 1/2$, $i = 1, 2$. Moreover, writing the density of θ as $f(\theta) = 1$ and the joint density function of signals and state as $f(s_1, s_2, \theta)$, it follows that $\Pr(s_1, s_2|\theta) = \frac{f(s_1, s_2, \theta)}{f(\theta)} = f(s_1, s_2, \theta)$. Thus, the joint distribution of senders' signals conditional on the state equals the unconditional joint density on signals and state.

Equilibrium. We study the weak Perfect Bayesian Nash Equilibria of the game described above (see, e.g., Mas-Colell, Whinston, and Green (1995)). At every such

equilibrium the messages $t_i(s_i)$ of senders $i = 1, 2$ and the action $y(t_1, t_2)$ of the receiver have the following properties

- $t_i(s_i)$ maximizes the expected utility of the sender i, i.e.

$$t_i(s_i) = \arg \max_{t_i \in \{0,1\}} \sum_{s_{-i} = \{0,1\}} \int_0^1 - (y(t_i, t_{-i}(s_{-i})) - \theta - b_i)^2 f(s_{-i}, \theta | s_i) \, d\theta$$

- $y(t_1, t_2)$ maximizes the expected utility of the receiver, i.e.

$$y(t_1, t_2) = \arg \max_{y \in \mathbb{R}} \int_0^1 - (y - \theta - b_R)^2 f(\theta | t_1, t_2) \, d\theta.$$

2.1 Truth-telling Equilibrium

As it is common in cheap talk games, multiple equilibria exist and, in particular, a babbling outcome in which no information is transmitted is always an equilibrium. Since, however, our interest is in the effect of correlation on the incentives to communicate, we focus on the truth-telling equilibrium in which $t_i(s_i) = s_i$, $i = 1, 2$.[2] In a truth-telling equilibrium the receiver correctly learns the signals of the senders. Letting $y_{s_1, s_2} \equiv y(s_1, s_2)$ to simplify notation, the utility maximizing action of the receiver after being truthfully informed about signals s_1 and s_2 by the senders, is

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\begin{aligned}
y_{s_1, s_2} &= \arg \max_{y \in \mathbb{R}} \int_0^1 -(y - \theta - b_R)^2 f(\theta | s_1, s_2) \, d\theta \\
&= b_R + \int_0^1 \theta f(\theta | s_1, s_2) \, d\theta \\
&= b_R + \mathrm{E}[\theta | s_1, s_2]
\end{aligned}
\tag{1}
$$

where

$$f(\theta | s_1, s_2) = \frac{\Pr(s_1, s_2 | \theta)}{\int_0^1 \Pr(s_1, s_2 | \theta) d\theta}.$$

Simple algebra yields

$$y_{0,0} = b_R + \frac{1 + k}{2(2 + k)}, \qquad y_{0,1} = y_{1,0} = b_R + \frac{1}{2}, \qquad y_{1,1} = b_R + \frac{3 + k}{2(2 + k)}. \tag{2}$$

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[2]We further notice that the truth-telling equilibrium Pareto dominates all other equilibria — see, e.g., Galeotti, Ghiglino, and Squintani (2013).

Notice that only the actions based on identical signals depend on k. To understand why, consider the interpretation of the joint distribution given above in terms of a compound lottery: when the receiver hears the same signal from both senders, she believes that with probability k they have acquired information from the same source. To the contrary, when the receiver hears different signals, she infers that the senders' information certainly comes from independent sources. Notice also that, as expected, $y_{0,0} < y_{0,1} < y_{1,1}$ for all $k \in (0,1)$.

We now study a sender's incentives to report truthfully the observed signal. In a truth-telling equilibrium sender i knows that the other sender reports $t_{-i} = s_{-i}$. He therefore reports signal s_i instead of the false signal $1 - s_i$ if

$$\sum_{s_{-i} \in \{0,1\}} \int_0^1 - \left(y_{s_i, s_{-i}} - \theta - b_i \right)^2 f\left(s_{-i}, \theta | s_i \right) d\theta \geq \tag{3}$$

$$\geq \sum_{s_{-i} \in \{0,1\}} \int_0^1 - \left(y_{1-s_i, s_{-i}} - \theta - b_i \right)^2 f\left(s_{-i}, \theta | s_i \right) d\theta,$$

which, substituting $f\left(s_{-i}, \theta | s_i \right) = f\left(\theta | s_i, s_{-i} \right) \Pr\left(s_{-i} | s_i \right)$ by Bayes' rule and integrating, becomes

$$\sum_{s_{-i} \in \{0,1\}} - \left(y_{s_i, s_{-i}} - \mathrm{E}_f \left[\theta | s_i, s_{-i} \right] - b_i \right)^2 \Pr\left(s_{-i} | s_i \right) \geq$$

$$\geq \sum_{s_{-i} \in \{0,1\}} - \left(y_{1-s_i, s_{-i}} - \mathrm{E}_f \left[\theta | s_i, s_{-i} \right] - b_i \right)^2 \Pr\left(s_{-i} | s_i \right).$$

Since $y_{s_i, s_{-i}} = b_R + \mathrm{E}_f \left[\theta | s_i, s_{-i} \right]$ from (1), condition (3) further simplifies to

$$\sum_{s_{-i} \in \{0,1\}} \Pr\left(s_{-i} | s_i \right) \left(y_{1-s_i, s_{-i}} - y_{s_i, s_{-i}} \right)^2 \geq$$

$$\geq 2 \left(b_i - b_R \right) \sum_{s_{-i} \in \{0,1\}} \Pr\left(s_{-i} | s_i \right) \left(y_{1-s_i, s_{-i}} - y_{s_i, s_{-i}} \right). \tag{4}$$

Notice now that $\Pr\left(s_{-i} | s_i \right) = \frac{\Pr(s_{-i}, s_i)}{\Pr(s_i)} = 2 \Pr\left(s_{-i}, s_i \right)$. We can thus substitute optimal actions using (2) and $\Pr\left(s_{-i}, s_i \right)$ using Table 1 in the truth-telling condition (4). It is then easy to show that, whenever $s_i = 0$, truth-telling requires

$$b_i - b_R \leq \frac{1}{8 + 4k},$$

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while, when $s_i = 1$, it requires

$$b_i - b_R \geq -\frac{1}{8 + 4k}.$$

The next proposition characterizes the truth-telling equilibrium with two senders.

Proposition 1. *Let the joint distribution function of signals conditional on the state be as in Table 1. Then a truth-telling equilibrium in which both senders report truthfully the observed signals exists if, and only if*

$$d_{iR} \leq \frac{1}{8 + 4k} \qquad i = 1, 2 \tag{5}$$

where $d_{iR} \equiv |b_i - b_R|$ and $k \in [0, 1)$. It follows that the maximal distance in preferences consistent with truthful information revelation decreases as the correlation between information sources increases.

To understand Proposition 1 it is useful to refer to the so-called *overshooting* effect — see Morgan and Stocken (2008) p.871 — which pins-down threshold (5). A sender's incentive to lie comes from his desire to drag the receiver's action closer to his bliss point. However, when the displacement in the receiver's action caused by a lie is large compared to the distance in preferences, her action may end up being even further away from the sender's bliss point than in case he reports the signal truthfully. This concern prevents senders whose preferences are sufficiently close to those of the receiver from lying.

Once the overshooting mechanism is understood, the intuition behind Proposition 1 is straightforward. The strength of the overshooting effect depends on how informative the signal is for the receiver. When k is large, the informative content of a message is low. Accordingly, the impact of each message on the receiver's action will be lower on average, reducing the risk of overshooting. This, in turn, increases the incentives to lie and makes truth-telling an optimal strategy only for senders with preferences close to those of the receiver.[3]

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[3]Notice that when $k = 0$, we have $d_{iR} \leq \frac{1}{8}$ corresponding to Corollary 1 in Galeotti, Ghiglino,

3   Many Informed Experts

We now generalize the analysis to the case of n senders, indexed by $i \in \{1, ..., n\}$. The structure of the game is unchanged: each sender i observes a binary signal, $s_i \in \{0, 1\}$, and his utility depends on the action $y \in \mathbb{R}$ taken by the receiver, on the state θ and on the individual preference parameter b_i. First players observe signals, then each sender independently reports a message $t_i \in \{0, 1\}$ to the receiver, who then takes action y based on the vector of n messages, $\mathbf{t}$. Utilities are loss quadratic: $U^R (y, \theta, b_R) = - (y - \theta - b_R)^2$ for the receiver and $U^i (y, \theta, b_i) = - (y - \theta - b_i)^2$ for sender i.

As before, the state of the world is known to be uniformly distributed, $\theta \sim U[0, 1]$. Conditional on the state, senders' signals are drawn from a joint distribution $\Pr(\mathbf{s}|\theta)$, where $\mathbf{s}$ denotes the n-dimensional vector of signals. We assume that each sender's signal is informative — i.e., $\Pr(s_i = 1|\theta)$ increases with θ. For the moment, we put no further restrictions on $\Pr(\mathbf{s}|\theta)$, which is generically a function of $2^n - n - 1$ independent parameters satisfying n non-negativity and an addition-to-one constraints.[4] In Section 3.1 we study the conditions (on preferences) for the existence of a truth-telling equilibrium in such general statistical model. In Section 3.2 we then apply these results to a specific generative process for signals, and discuss the role of correlation for truth-telling.

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and Squintani (2013). Moreover, the general principle that more informative signals sustain the truth-telling equilibrium for a larger distance in preferences is behind other results in the literature such as Morgan and Stocken (2008) and, more recently, Galeotti, Ghiglino, and Squintani (2013) — all featuring conditionally independent signals.

[4]See Bahadur (1961) for a useful representation theorem on the joint probability distribution of n dichotomous items.

3.1 Truth-telling Equilibrium

A weak Perfect Bayesian Nash Equilibrium (PBNE) of this game is defined in exactly the same way as for the case of two senders. Let $\mathbf{t}_{-i}$ denote the the vector of all messages sent by senders except message t_i. An equilibrium of this game is a strategy $t_i(s_i)$ for each sender i — denoted $\mathbf{t}(\mathbf{s})$ in vector form — and a strategy $y(\mathbf{t})$ for the receiver, such that

- $t_i(s_i)$ maximizes the expected utility of sender i

$$t_i(s_i) = \arg \max_{t_i \in \{0,1\}} \sum_{\mathbf{t}_{-i} \in \{0,1\}^{n-1}} \int_0^1 -(y(\mathbf{t}_{-i}(\mathbf{s}_{-i}), t_i) - \theta - b_i)^2 f(\mathbf{s}_{-i}, \theta | s_i) \, d\theta.$$

- $y(\mathbf{t})$ maximizes the expected utility of the receiver, i.e.

$$y(\mathbf{t}) = \arg \max_{y \in \mathbb{R}} \int_0^1 -(y - \theta - b_R)^2 f(\theta | \mathbf{t}) \, d\theta$$

We focus on truth-telling equilibrium, in which $\mathbf{t}(\mathbf{s}) = \mathbf{s}$. Let $y(\mathbf{s}) \equiv y_\mathbf{s}$ be the utility maximizing action of the receiver after observing the vector of signals $\mathbf{s}$. It is immediate to verify that

$$y_\mathbf{s} = b_R + \int_0^1 \theta f(\theta | \mathbf{s}) \, d\theta = b_R + \mathrm{E}[\theta | \mathbf{s}].$$

Turning to senders' behavior, at a truth-telling equilibrium sender i is willing to report truthfully his signal if

$$\sum_{\mathbf{s}_{-i} \in \{0,1\}^{n-1}} \int_0^1 -(y_{s_i, \mathbf{s}_{-i}} - \theta - b_i)^2 f(\mathbf{s}_{-i}, \theta | s_i) \, d\theta$$

$$\geq \sum_{\mathbf{s}_{-i} \in \{0,1\}^{n-1}} \int_0^1 -(y_{1-s_i, \mathbf{s}_{-i}} - \theta - b_i)^2 f(\mathbf{s}_{-i}, \theta | s_i) \, d\theta.$$

Expanding squares and rearranging terms, the above inequality simplifies to

$$\sum_{\mathbf{s}_{-i} \in \{0,1\}^{n-1}} \Pr(\mathbf{s}_{-i} | s_i) \Delta^2(\mathbf{s}_{-i} | s_i) \geq 2(b_i - b_R) \sum_{\mathbf{s}_{-i} \in \{0,1\}^{n-1}} \Pr(\mathbf{s}_{-i} | s_i) \Delta(\mathbf{s}_{-i} | s_i) \qquad (6)$$

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where $\Delta(\mathbf{s}_{-i}|s_i) \equiv y_{1-s_i,\mathbf{s}_{-i}} - y_{s_i,\mathbf{s}_{-i}}$ measures the displacement of the receiver's optimal action following an undetected lie from sender i reporting $1-s_i$ instead of the observed signal s_i.

The next proposition characterizes a truth-telling equilibrium for the case with n senders. To simplify notation, we will denote the "expected displacement" appearing on the RHS of (6) as

$$\mathrm{E}\left[\Delta\left(\mathbf{s}_{-i}|s_i\right)\right] \equiv \sum_{\mathbf{s}_{-i}\in\{0,1\}^{n-1}} \Pr\left(\mathbf{s}_{-i}|s_i\right)\Delta\left(\mathbf{s}_{-i}|s_i\right). \tag{7}$$

We use a similar notation for the expected value of the squared displacements, appearing on the LHS of (6).

Proposition 2. *Let* $\Pr(\mathbf{s}|\theta)$ *be a joint distribution of* n *informative signals conditional on the state* θ. *A truth-telling equilibrium exists if and only if* $d_{iR} \leq d_i^*(s_i)$ *for all* i *and* s_i *such that*

$$(b_i - b_R) \cdot \mathrm{E}\left[\Delta\left(\mathbf{s}_{-i}|s_i\right)\right] > 0,$$

with

$$d_i^*(s_i) \equiv \frac{1}{2}\frac{\mathrm{E}\left[\Delta^2\left(\mathbf{s}_{-i}|s_i\right)\right]}{\left|\mathrm{E}\left[\Delta\left(\mathbf{s}_{-i}|s_i\right)\right]\right|}. \tag{8}$$

We first note that Proposition 1 is a special instance of Proposition 2 for the case of two senders and a symmetric information structure with correlation parameter k. As we did for Proposition 1, we can understand Proposition 2 in terms of the overshooting effect. A key role is played by the expected displacement of the receiver's action following sender i's deviation from a truth-telling equilibrium, appearing at the denominator of (8).

To fix ideas, suppose that sender i has observed $s_i = 0$. Misreporting his signal, i induces the displacement $\Delta(\mathbf{s}_{-i}|0)$. If $\Pr(\mathbf{s}|\theta)$ is (conditionally) independent, then $\Delta(\mathbf{s}_{-i}|0)$ is positive for all $\mathbf{s}_{-i}$: the receiver, who believes sender i's word, revises upward her expectations on θ because the signal of each sender is informative and she believes the state is larger when she hears $t_i = 1$ rather than $t_i = 0$.[5] This is

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[5] As an illustration, if signals are i.i.d. with $\Pr(s = 1|\theta) = \theta$, then $\Pr(\mathbf{s}|\theta) = B(n,\theta)$.

what we referred to in the introduction as the *direct effect* of information provision. By the same logic, $\Delta(\mathbf{s}_{-i}|1)$ is negative for all $\mathbf{s}_{-i}$. Hence, if signals are independent, the expected displacement is never null, it is positive when $s_i = 0$ and negative when $s_i = 1$.[6] As we have shown in Section 2.1, these results also characterize the case of correlated signals with two informed senders. There, the displacement induced by i reporting $t_i = 1$ ($t_i = 0$) when observing $s_i = 0$ ($s_i = 1$) is positive (negative), independently of j's signal.[7]

Inspection of expression (8) tells us that both in the case of independent signals, and in the case of correlated signals with two senders, truth-telling requires closer preferences the smaller the expected displacement $\mathrm{E}[\Delta(\mathbf{s}_{-i}|s_i)]$. In fact, each term of the numerator of (8) tends to zero faster than the corresponding term in the denominator, implying that the truth-telling condition becomes tighter when the expected displacement is smaller on average.

The key insight of this and the next sections is that the above conclusion does not extend to the case of $n > 2$ senders and arbitrary probability distributions $\Pr(\mathbf{s}|\theta)$. When signals are correlated, the displacement following a misreport by sender i may take opposite sings depending on the signals reported by the other senders. Intuitively, this happens because the way in which the receiver updates her belief on the state using signals $\mathbf{s}_{-i}$ depends, through the correlation structure, on the message t_i. This is what we called the *indirect effect* of information provision on the receiver's belief about the state. When displacements can take different signs, the numerator in (8), being a sum of squares, remains bounded away from zero, but the expected displacement appearing at the denominator may vanish, being the sum of negative and positive terms. In such cases, the incentives to report the truth — i.e., the threshold $d_i^*(s_i)$ — can grow without bounds.

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[6]Although it can be very small when n is large. In fact, the information reported by a single sender adds less to the information aggregated by the receiver when the number of senders is large.

[7]Here the indirect effect is at play, as commented later, but the direct one prevails.

3.2 A Process with Correlated Signals

In this section we study a specific generative process for binary signals, which naturally extends the structure of the (unique) joint probability distribution for the two-senders case characterized in Lemma 1 — and coincides with it when $n = 2$. In particular, signals are either generated by a binomial distribution with probability $1 - k$ or, with complementary probability, by a perfectly correlated process. The probability that signals come from the correlated process, k, is indeed the *ex-ante* Pearson pairwise correlation index of signals. As we shall see, this distribution naturally generates an updating process in which the expected displacements following sender i's misreport may carry opposite sings depending on the signals reported by the other senders.

The joint probability distribution of signals conditional on the state is

$$\Pr\left(\mathbf{s}|\theta\right) = \begin{cases} \Pr\left(\mathbf{1}_n|\theta\right) = \theta^n \left(1 - k\right) + \theta k \\ \Pr\left(\mathbf{0}_n|\theta\right) = \left(1 - \theta\right)^n \left(1 - k\right) + \left(1 - \theta\right) k \\ \Pr\left(\mathbf{1}_l, \mathbf{0}_{n-l}|\theta\right) = \theta^l (1 - \theta)^{n-l} \left(1 - k\right) \quad \text{if} \quad 0 < l < n \end{cases} \tag{9}$$

where $\mathbf{1}_x$ (resp. $\mathbf{0}_x$) is a vector of ones (resp. zeros) of dimension x and $(\mathbf{1}_x, \mathbf{0}_y)$ is a vector of $x + y$ signals containing x ones and y zeros in any ordering. Like the distribution of Table 1, the joint distribution (9) is a mixture between two distributions: a binomial distribution $B(n, \theta)$ taken with weight $1 - k$ and a degenerate joint distribution corresponding to a single Bernoulli experiment with success probability equal to θ, taken with weight k. The process generating $\Pr\left(\mathbf{s}|\theta\right)$ is then one in which senders either collect information from n independent sources with probability $1 - k$ or, with probability k, they collect information from perfectly correlated sources — or, equivalently, the same source. As before, all information sources are such that $\Pr\left(s_i = 1|\theta\right) = \theta$, $i \in \{1, .., n\}$.

It is useful to represent this process as a compound lottery (Figure 1). Note that, by construction, a mixed sequence of signals can only occur when signals come from
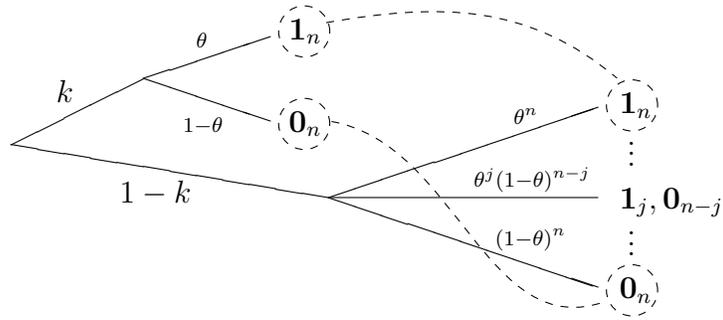
Figure 1: Signals-generating process as a compound lottery

independent sources, which happens with probability $1 - k$. To the contrary, a full string of zeros or ones may be the result of a series of independent draws as well as the outcome of players observing the same source — hence, the dotted lines indicating "information sets". As a consequence, the k-weighted term does not appear in the third line of (9), because the probability of a mixed sequence of signals coming from the same source — top branch of the tree — is zero.

The simple structure of the generative process underlying (9) allows to easily compute conditional probabilities applying Bayes' rule along the tree depicted in Figure 1. Formally, given the non-negative integers a, b, c and d with $a+b+c+d \leq n$, define the conditional probability

$$\Pr\left(\mathbf{1}_a, \mathbf{0}_b | \mathbf{1}_c, \mathbf{0}_d, \theta\right) = \frac{\Pr\left(\mathbf{1}_{a+c}, \mathbf{0}_{b+d} | \theta\right)}{\Pr\left(\mathbf{1}_c, \mathbf{0}_d | \theta\right)}$$

as the probability that a string of a ones and b zeros is observed given a string of c ones and d zeros has been observed and the state is θ. Using (9) we obtain

$$\Pr\left(\mathbf{1}_a, \mathbf{0}_b | \mathbf{1}_c, \mathbf{0}_d, \theta\right) = \begin{cases} \theta^a \left(1-\theta\right)^b & \text{if} \quad c \neq 0 \wedge d \neq 0 \\ \frac{\theta^{a+c}(1-\theta)^b(1-k)+\mathbb{1}(b=0)\theta k}{\theta^c(1-k)+\theta k} & \text{if} \quad c \neq 0 \wedge d = 0 \\ \frac{\theta^a(1-\theta)^{b+d}(1-k)+\mathbb{1}(a=0)(1-\theta)k}{(1-\theta)^d(1-k)+(1-\theta)k} & \text{if} \quad c = 0 \wedge d \neq 0 \end{cases} \quad (10)$$

where $\mathbb{1}(\cdot)$ is an indicator function taking value one when the argument is true and zero otherwise. The conditional probabilities above are derived in Lemma 3 in the Appendix.[8] The first line of (10) has a straight interpretation: once (at least) two

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[8]The lemma characterizes the joint distribution of any subset of $(n - l)$ signals, $\Pr(\mathbf{s}|\theta)$ with

different signals have been observed ($c \neq 0$ and $d \neq 0$) it is immediately inferred that the observed combination lays on the lower branch of the tree where the information sources are independent, hence the simple binomial expression. To the contrary, when either $c = 0$ or $d = 0$, the observed signals, $(\mathbf{1}_c, \mathbf{0}_d)$, do not allow to determine which branch of the tree the system is on. The conditional probability accounts for this uncertainty by incorporating k as reported in lines two and three of (10).

The distribution just introduced has several useful properties, which make it tractable and intuitive. First, it is symmetric in the sense of invariant to index permutations — i.e., $\Pr(s_1, ..., s_n) = \Pr(s_{i_1}, ..., s_{i_n})$ for all permutations $i_1, ..., i_n$. Second, it has the property that marginal probabilities conditional on the state are equal to the state for each sender, that is $\Pr(s_i = 1|\theta) = \theta$. This is equivalent to say that each sender's message is equally informative about the state. Third, it is not difficult to show that, as in the two-senders case, k denotes the Pearson's correlation coefficient of the joint probability distribution of any two signals.[9] Thus, the interpretation of k is unaltered in the model with n senders.[10]

3.2.1 Truth-telling and the Effect of Correlation

We now characterize the truth-telling equilibrium under the joint probability distribution (9). The optimal action taken by the receiver when she believes the n truthful

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$\mathbf{s} \in \{0, 1\}^{n-l}$ and $0 < l < n$.

[9]To see this, apply Lemma 3 in the Appendix to any couple of signals, s_i and s_j: the lemma shows that, marginalizing over the remaining signals $\mathbf{s}_{-\{i,j\}}$, the joint distribution $\Pr(s_i, s_j|\theta)$ is exactly as the one in Table 1.

[10] Notice that the correlation structure of (9) is clearly more complex than that. In particular, the correlation coefficients of order $m = 3, ..., n$ can be traced back to the marginal θ and to the pairwise correlation k as follows

$$r_{(m)} = k\theta (1 - \theta) \frac{(-1)^m \theta^{m-1} + (1-\theta)^{m-1}}{\left(\sqrt{\theta(1-\theta)}\right)^m} \qquad m = 3, ..., n.$$

messages is $E(\theta|\mathbf{s})$; for the current distribution it takes the following values

$$y_{\mathbf{0}_n} = \frac{6 + k(n-1)(n+4)}{3(n+2)(2-k+kn)}, \quad y_{\mathbf{1}_l,\mathbf{0}_{n-l}} = \frac{1+l}{n+2}, \quad y_{\mathbf{1}_n} = \frac{2(n+1)(3-k+kn)}{3(n+2)(2-k+kn)}. \quad (11)$$

These expression are obtained in exactly the same way as for the case with two senders. We refer to the procedure illustrated in the proof of Proposition 1 and omit their full derivation here.

Note that, while $y_{\mathbf{1}_l,\mathbf{0}_{n-l}}$ does not depend on k, $y_{\mathbf{1}_n}$ decreases and $y_{\mathbf{0}_n}$ increases with k: by reducing the informative content of each signal, correlation reduces the distance between the optimal actions following two strings of identical signals. It can be easy shown that as k tends to 1 (perfectly correlated signals), $y_{\mathbf{0}_n}$ tends to $\frac{1}{3}$ and $y_{\mathbf{1}_n}$ to $\frac{2}{3}$, which are, respectively, the posteriors on $E(\theta|s)$ given a single zero or one signal. Finally, when k approaches 0 (independent signals), $y_{\mathbf{0}_n}$ and $y_{\mathbf{1}_n}$ tend to $\frac{1}{n+2}$ and $\frac{n+1}{n+2}$ respectively which, as expected, tend to 0 and 1 as n grows large.

Before presenting the truth-telling conditions for the n senders, we reconsider the expected displacement $E[\Delta(\mathbf{s}_{-i}|s_i)]$ — see expression (7) — in light of the probability distribution (9). These terms will play a key role in our result of Proposition 3. We first note that, by definition

$$-\Delta(\mathbf{0}_{n-1}|0) = \Delta(\mathbf{0}_{n-1}|1) \qquad \text{and} \qquad -\Delta(\mathbf{1}_{n-1}|0) = \Delta(\mathbf{1}_{n-1}|1).$$

Moreover, from (11)

$$\Delta(\mathbf{0}_{n-1}|0) = \frac{6 - k(2 - 3n + n^2)}{3(2 + k(n-1))(2+n)} = -\Delta(\mathbf{1}_{n-1}|1), \quad (12)$$

$$\Delta(\mathbf{0}_{n-1-l}, \mathbf{1}_l|0) = \frac{1}{2+n} = -\Delta(\mathbf{0}_{n-1-l}\mathbf{1}_l|1) \qquad (l \neq 0, n-1). \quad (13)$$

It can be easily checked that $\Delta(\mathbf{0}_{n-1}|0) > 0$ for all admissible values of k when $n \leq 4$, while $\Delta(\mathbf{0}_{n-1-l}, \mathbf{1}_l|0) > 0$ for all values of n. It follows that the expected displacement $E[\Delta(\mathbf{s}_{-i}|0)]$ is strictly positive (and $E[\Delta(\mathbf{s}_{-i}|1)]$ is strictly negative) for all $n \leq 4$. When $n > 4$ the displacement $\Delta(\mathbf{0}_{n-1}|0)$ becomes negative for k large enough — see (12).[11] The reason why the receiver would react to the report of $t_i = 1$

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[11]Note that $\Delta(\mathbf{0}_{n-1}|0)$ in (12) is monotone decreasing in both n and k.

by decreasing its action is to be found in the way she updates her prior on θ. While a *low* report by sender i, in line with the other $n-1$ reports, is consistent with both independent and perfectly correlated signals, a *high* report induces the receiver to believe that all the n signals come from independent sources. Note that the effect of this shift in the receiver's posterior depends on both k and n. First, the larger k, the more salient is the effect of the update on the action; second, the larger n, the larger the number of *low* signals that the receiver interprets as independent. This explains why a negative displacement $\Delta\left(\mathbf{0}_{n-1}|0\right)$ requires both a large enough k and numerous enough senders. A similar logic applies to $\Delta\left(\mathbf{1}_{n-1}|1\right)$.

The next lemma summarizes these observations, and shows that there exists a specific value of the correlation parameter k such that the positive and the negative terms in the expected displacements formula exactly balance.

Lemma 2. *Let* $\Pr\left(\mathbf{s}|\theta\right)$ *be defined by* (9). *If* $n > 4$, *there exists a unique value* $\bar{k}(n)$ *of the pairwise correlation for which the expected displacement is equal to zero*

$$\bar{k}(n) \equiv \tfrac{1}{2(1+n)}\left(1 + \sqrt{1 + 24\tfrac{n(1+n)}{(n-2)(n-1)}}\right).\tag{14}$$

If $n \leq 4$, *the expected displacement is always different from zero.*

We are now ready to specialize Proposition 2 to the probability distribution (9).

Proposition 3. *Let* $\Pr\left(\mathbf{s}|\theta\right)$ *be defined by* (9). *If* $k \neq \bar{k}(n)$ *then a truth-telling equilibrium exists if and only if* $d_{iR} \leq d^*\left(n,k\right)$ *for all* i, *with*

$$d^*\left(n,k\right) \equiv \left|\tfrac{36n+k(n-2)(n-1)(12+k(-17+k(n-5)(n-1)(n+1)+n(2n-9)))}{6(2+k(n-1))(2+n)(6n-k(n-2)(n-1)(k(n+1)-1))}\right|.$$

If $k = \bar{k}(n)$ *a truth-telling equilibrium exists for any* d_{iR}, *for all* i.

Figure 2 illustrates the finding of Proposition 3 by plotting the maximum distance in preferences allowing for truth-telling as a function of k, for different values of n. In particular, the black lines define the maximum distance between sender i's and the receiver's preference parameters such that sender i truthfully reports $s_i = 0$. The
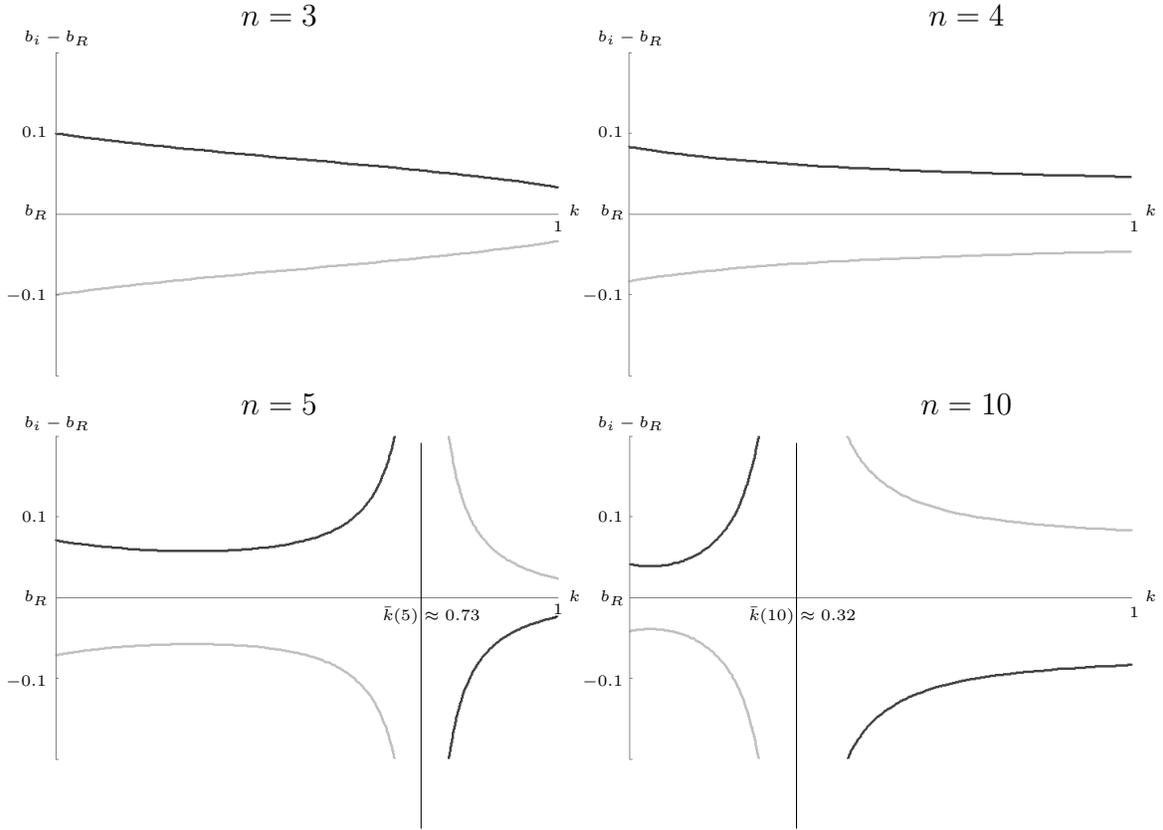
Figure 2: Threshold bands for sender i at some truth-telling equilibria.

gray lines define instead the maximum distance in preferences such that sender i truthfully reports $s_i = 1$. We focus on the qualitative patterns of the truth-telling thresholds. The top panels refer to cases with $n \leq 4$. Here a sender with $b_i > b_R$ $(b_i < b_R)$ never lies on observing $s_i = 1$ $(s_i = 0)$, no matter the correlation level k. This is a direct consequence of the fact that when $n \leq 4$ all the terms in the expected displacement formula in (7) are negative. The bottom panels refers instead to cases with $n > 4$. Here, as we argued in the discussion prior to Lemma 2, the various terms of the expected displacement carry opposite signs for k large enough, and the value of the expected displacement tends zero as k approaches $\bar{k}(n)$. When this happens, the truth-telling condition (6) is always satisfied.

As k increases beyond $\bar{k}(n)$, the expected displacement $\mathrm{E}[\Delta\left(\mathbf{s}_{-i}|0\right)]$ becomes negative, meaning that an upward-biased sender i — i.e., with $b_i > b_R$ — has no incen-

tive to lie on observing $s_i = 0$. However, beyond $\bar{k}(n)$ the expected displacement $\mathrm{E}[\Delta\left(\mathbf{s}_{-i}|1\right)]$ becomes positive, meaning that a sender with a positive bias in preferences may now have an incentive to report $t_i = 0$ on observing $s_i = 1$. This counter-intuitive incentive is due to the belief of independent signals that a string of mixed reports induces on the receiver. Reporting a *low* signal given a uniform string of *high* reports by the other senders induces an upward displacement of the receiver's action; the same would result from the report of a *low* signal given a uniform string of *low* reports, since such misreport would avoid the updating of the receiver's beliefs.

By considering several senders, Proposition 3 brings in new insights to our understanding of information transmission. While with two informed senders (and, in our specific process, with $n \leq 4$) the correlation of signals always weakens the incentives of each sender to truthfully reveal private information (consistently with Proposition 1), with more senders ($n > 4$ in our example) correlation has non-monotonic effects on the incentives to truthfully reveal information. Strings of mixed reports are interpreted (in equilibrium) as evidence of independent sources. When reporting a signal which is likely to be not in line with all the other reports, each sender takes this into account and anticipates the possibly averse effect on the receiver's action, facing, as a result, weaker incentives to misreport. In this sense, correlation *disciplines* a sender's reporting behavior by increasing the relative profitability of consistent reports. Due to the mechanics discussed above, this discipline is stronger when k is closer to $\bar{k}(n)$.

In the next final proposition we qualify the result of Proposition 3 by studying the limiting behavior of both the truth-telling threshold $d^*(n, k)$ and the critical correlation level $\bar{k}(n)$.

Proposition 4. *Let $d^*(n, k)$ be the threshold derived in Proposition 3 and $\bar{k}(n)$ be defined by* (14). *Then:*

$$1. \ \lim_{k \to 0} \frac{\partial d^*(n,k)}{\partial k} < 0, \qquad 2. \ \lim_{n \to \infty} \bar{k}(n) = 0, \qquad 3. \ \lim_{n \to \infty} d^*(n, k) = \tfrac{1}{6}.$$

The first point of Proposition 4 tells us that a small amount of correlation always tightens the requirements for truth-telling relative to independent signals. Hence,

little correlation is bad for information transmission. The second and third points provide insights on the incentives to reveal information when n grows large. First, as the number of senders grows, the indirect effect, responsible for the non-monotone pattern of the threshold, grows in magnitude. This is the result of the increasing size of the displacements following a revision in the receiver's beliefs, and of the relative odds of $n-1$ uniform reports which does not vanish as n grows because of the correlation of signals. Second, the impact of any individual message on the receiver's action is non negligible even for n large, implying that $d^*(n,k)$ does not tend to zero. This is a remarkable property, due to the correlation of signals, which must be contrasted with the traditional result for the case of independent signals, where truth-telling equilibria do not exist when the number of informed senders grows large (see Morgan and Stocken (2008) and, more recently, Galeotti, Ghiglino, and Squintani (2013))

3.3 Robustness and Open Questions

The example provided in Section 3.2 is neat and easy to interpret. However, it is worth emphasizing that the main result made so clear by Proposition 3 — i.e., the non-monotonic pattern of truth-telling incentives with respect to correlation — is by no means an artifact of the joint distribution (9). Using the representation theorem by Bahadur (1961) and mathematical software it is not difficult to construct other joint distributions featuring similar non-monotonic behavior. For instance, one can show that the non-monotonic shape of truth-telling thresholds holds for a symmetric joint distribution of $n = 3$ signals featuring marginals equal to θ, pairwise correlation equal to $k \in [0,1)$ and three-ways correlation equal to $k \cdot r_{(3)}$ with $r_{(3)}$ defined in Footnote 10. In this case, as in the top panels of Figure 2, incentives never revert, but the non-monotonic behavior neatly observed in the bottom panels of the same figure kicks-in as soon as $n = 3$.

Finally, we close with a comment on the meaning of the term "correlation" in our environment. In our analysis, we have mainly referred to the *pairwise* correlation,

which we know how to interpret and understand in intuitive terms. However, it is clear from the above discussion — and from Bahadur (1961) — that, when information is aggregated from more than two sources, the incentives to communicate truthfully depend on the entire *correlation structure* of the generative process of signals. We leave to future research a thorough analysis of the relationship between truth-telling incentives and the entire correlation structure of general distributions of signals.

4   Conclusion

We have studied the role of conditional correlation of private information in shaping truth-telling incentives in a cheap talk game. We have found that in a model with only two senders, correlation of the senders' signals reduces the interval of preference biases that support truth-telling as an equilibrium. We have then turned to information transmission when there are n informed senders. First, we have characterized general truth-telling conditions for a generic joint distribution of possibly correlated binary signals. We have argued that in this case correlation may facilitate as well as impair truthful communication. To illustrate the point, we have studied a specific generative process, in which signals either come from independent or from perfectly correlated sources. The *ex-ante* probability of the latter event is shown to measure the pairwise correlation index of the overall process. The key feature of this example is the fact that the configuration of reported signals carries both a (direct) informational content about the state of the world and an (indirect) informational content about the process from where signals are drawn. In the proposed process, this indirect channel takes a very stark form, revealing to the receiver that signals come from independent sources whenever they are not perfectly aligned. We have shown that this indirect effect is responsible for the non-monotonic effect of correlation on truth-telling — see Section 3.2.1 — and for the unrestricted possibility to transmit information at specific levels of correlation and number of experts.

While an exhaustive analysis of all possible correlation structures is beyond the scope of this paper, we believe our contribution points to a fundamental and previously unnoticed role of correlation for information aggregation and transmission. Misreporting a signal that is in line with the rest of the reports increases the perceived degree of independence of all transmitted information and, with it, the elasticity of the receiver's decision. As we have seen, this can shift the receiver's decision in the direction opposite to the misreport, a 'discipline' effect that favors truth-telling.

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Appendix

Proof of Lemma 1. To prove the lemma we use the representation theorem by Bahadur (1961). Applying Proposition 1 (see page 159 therein), for any pair of signals $\mathbf{s} = (s_1, s_2)$ we can write the (conditional) probability of $\mathbf{s}$ as the product of the probability in case s_1 and s_2 were independent and a 'correction term', that is

$$\Pr(\mathbf{s}|\theta) = \Pr_{[\mathrm{I}]}(\mathbf{s}|\theta) f(\mathbf{s}|\theta),$$

with

$$\Pr_{[\mathrm{I}]}(\mathbf{s}|\theta) = \prod_{i=1}^{2} \theta^{s_i} (1-\theta)^{1-s_i} \qquad \text{and} \qquad f(\mathbf{s}|\theta) = 1 + r_{12} z_1 z_2$$

where $z_i = \frac{s_i - \theta}{\sqrt{\theta(1-\theta)}}$, $i = 1, 2$ and $r_{12} \equiv \mathrm{E}(z_1 z_2)$ is the correlation coefficient between s_1 and s_2 according to $\Pr(\mathbf{s}|\theta)$. Let's denote r_{12} with k. Then, applying the above representation and taking into account symmetry — i.e., $\Pr(1, 0|\theta) = \Pr(0, 1|\theta)$ — we can write

$$\Pr(0, 0|\theta) = (1-\theta)^2 \left(1 + k\frac{\theta^2}{\theta(1-\theta)} \right) = (1-\theta)k + (1-\theta)^2(1-k)$$

$$\Pr(0, 1|\theta) = \theta(1-\theta) \left(1 + k\frac{-\theta(1-\theta)}{\theta(1-\theta)} \right) = \theta(1-\theta)(1-k)$$

$$\Pr(1, 1|\theta) = \theta^2 \left(1 + k\frac{(1-\theta)^2}{\theta(1-\theta)} \right) = \theta k + \theta^2(1-k)$$

The system above of three equations in three unknowns — $\Pr(\mathbf{s}|\theta)$ for $\mathbf{s} = (0, 0), (0, 1), (1, 1)$ — characterizes the unique (symmetric) joint probability distribution for the case with two players.

Finally, it is easy to check that k is the Pearson correlation coefficient for the distribution $\Pr(\mathbf{s}|\theta)$ — i.e., $k = \frac{Cov(s_1, s_2)}{\sigma_{s_1} \sigma_{s_2}}$. In fact,

$$\mathrm{E}(s_i) = 0 \cdot \Pr(0|\theta) + 1 \cdot \Pr(1|\theta) = \theta \qquad i = 1, 2$$

and, because $\mathrm{E}(s_i^2) = \mathrm{E}(s_i)$,

$$\sigma_{s_i} = \sqrt{\mathrm{E}(s_i^2) - \mathrm{E}(s_i)^2} = \sqrt{\theta(1-\theta)} \qquad i = 1, 2$$

Finally, the covariance between signals i and j is

$$
\begin{aligned}
Cov\left(s_i, s_j\right) &= \mathrm{E}\left(s_i s_j\right) - \mathrm{E}\left(s_i\right)\mathrm{E}\left(s_j\right) \\
&= \Pr\left(1, 1|\theta\right) - \theta^2 \\
&= \theta\left(1-\theta\right)k.
\end{aligned}
$$

It follows that

$$
\frac{Cov\left(s_i, s_j\right)}{\sigma_{s_i}\sigma_{s_j}} = k.
$$

∎

Proof of Proposition 1 (derivation of $f\left(\theta|s_i, s_j\right)$ and y_{s_i, s_j}). The main steps of the proof of Proposition 1 are in the text. To the reader's convenience we derive here $f\left(\theta|s_i, s_j\right)$ and y_{s_i, s_j}. Applying Bayes rule and noticing that $f\left(\theta\right) = 1$ because $\theta \sim U\left(0, 1\right)$, the general expression for the conditional distribution of θ given a pair of signals $\left(s_i, s_j\right)$ is

$$
\begin{aligned}
f\left(\theta|s_i, s_j\right) &= \frac{\Pr\left(s_i, s_j|\theta\right)f\left(\theta\right)}{\int_0^1 \Pr\left(s_i, s_j|\theta\right)f\left(\theta\right)d\theta}, \\
&= \frac{\Pr\left(s_i, s_j|\theta\right)}{\int_0^1 \Pr\left(s_i, s_j|\theta\right)d\theta}.
\end{aligned}
$$

Using Table 1 we calculate the conditional density functions given all possible pairs of signals

$$
\begin{aligned}
f\left(\theta|s_i = 0, s_j = 0\right) &= \frac{6}{2+k}\left[\left(1-\theta\right)^2 + \theta\left(1-\theta\right)k\right], \\
f\left(\theta|s_i = 0, s_j = 1\right) &= f\left(\theta|s_i = 1, s_j = 0\right) = 6\theta(1-\theta), \qquad\text{(A1)} \\
f\left(\theta|s_i = 1, s_j = 1\right) &= \frac{6}{2+k}\left[\theta^2 + \theta\left(1-\theta\right)k\right].
\end{aligned}
$$

The optimal action of the receiver when she observes the pair of signals $\left(s_i, s_j\right)$ is

$$
\begin{aligned}
y_{s_i, s_j} &= b_j + \mathrm{E}\left[\theta|s_i, s_j\right], \\
&= b_j + \int_0^1 \theta f\left(\theta|s_i, s_j\right)d\theta,
\end{aligned}
$$

which, using (A1), yields (2). ∎

Proof of Proposition 2. The generic truth-telling condition for sender i who has observed a signal s_i is derived in the text. Using (7) condition (6) can be rewritten as

$$\mathrm{E}\left[\Delta^2\left(\mathbf{s}_{-i}|s_i\right)\right] \geq 2\left(b_i - b_R\right)\mathrm{E}\left[\Delta\left(\mathbf{s}_{-i}|s_i\right)\right], \tag{A2}$$

where the expectations of $\Delta(\mathbf{s}_{-i}|s_i) = y_{1-s_i,\mathbf{s}_{-i}} - y_{s_i,\mathbf{s}_{-i}}$ and its squares are taken over the probability $\Pr(\mathbf{s}_{-i}|s_i)$. At a truth-telling equilibrium the condition must hold for both $s_i = 0, 1$ for each $i = 1, ..., n$.

Suppose first that $\mathrm{E}\left[\Delta\left(\mathbf{s}_{-i}|0\right)\right] = \mathrm{E}\left[\Delta\left(\mathbf{s}_{-i}|1\right)\right] = 0$, for some sender i. Then it is clear from (A2) that sender i has no incentives to lie. Consider now the case in which, for sender i and signal s_i, $\mathrm{E}\left[\Delta\left(\mathbf{s}_{-i}|s_i\right)\right] \neq 0$. If the RHS of (A2) is negative, then again i has no incentive to report the false message $t_i = 1 - s_i$. If instead the RHS is positive, then, recalling that $d_{iR} \equiv |b_i - b_R|$, condition (A2) is certainly violated for sufficiently large d_{iR}. Hence, because $|ab| = |a||b|$ for real a and b, it is immediate to verify that in this case (A2) is satisfied if and only if

$$d_{iR} \leq \frac{1}{2}\frac{E\left[\Delta^2\left(\mathbf{s}_{-i}|s_i\right)\right]}{\left|E\left[\Delta\left(\mathbf{s}_{-i}|s_i\right)\right]\right|}.$$

The proof is completed once we define the RHS of the above inequality as $d_i^*\left(s_i\right)$ — to underline its dependency on the sender i's identity and the observed signal s_i. ∎

The next lemma provides an expression for the marginal probability of any subset of signals given the joint probability distribution (9) conditional on the state θ. It also shows that the probability structure of a generic subset of signals is "regular" in that it follows the behavior of a standard binomial probability distribution.

Lemma 3. *Assume that the joint probability distribution $\Pr\left(\mathbf{s}|\theta\right)$ is given by (9) and consider a $(n - l)$-tuple of signals, $0 < l < n$. Its joint probability distribution (conditional on the state θ) is*

$$\begin{cases} \Pr\left(\mathbf{1}_{n-l}|\theta\right) = \theta^{n-l}\left(1 - k\right) + \theta k \\[2mm] \Pr\left(\mathbf{0}_{n-l}|\theta\right) = \left(1 - \theta\right)^{n-l}\left(1 - k\right) + \left(1 - \theta\right)k \\[2mm] \Pr\left(\mathbf{1}_j, \mathbf{0}_{n-l-j}|\theta\right) = \theta^j(1 - \theta)^{n-l-j}\left(1 - k\right) \quad if \quad 0 < l < n \end{cases}$$

where $\Pr\left(\mathbf{1}_j, \mathbf{0}_{n-l-j}|\theta\right)$ *is the probability of an unsorted sequence of signals containing* j *ones,* $n - l - j$ *zeros and* l *other one or zero signals.*

Proof of Lemma 3. Using the notation introduced in Section 3 and letting $\mathbf{s}_l$ be an l-dimensional vector of zero or one signals, we can write

$$
\begin{aligned}
\Pr\left(\mathbf{0}_{n-l}|\theta\right) &= \sum_{\mathbf{s}_l \in \{0,1\}^l} \Pr\left(\mathbf{0}_{n-l}, \mathbf{s}_l|\theta\right), \\
&= \Pr\left(\mathbf{0}_n|\theta\right) + \sum_{j=0}^{l-1} \Pr\left(\mathbf{0}_{n-l}, \mathbf{0}_j, \mathbf{1}_{l-j}|\theta\right), \\
&= \underbrace{(1-\theta)\,k + (1-\theta)^n\,(1-k)}_{\Pr(\mathbf{0}_n|\theta)} \; + \\
&\qquad + \underbrace{(1-\theta)^{n-l}\,(1-k)\sum_{j=0}^{l-1} \frac{l!}{(l-j)!j!}\,(1-\theta)^j\,\theta^{l-j}}_{\sum_{j=0}^{l-1}\Pr\left(\mathbf{0}_{n-l},\mathbf{0}_j,\mathbf{1}_{l-j}|\theta\right)}, \\
&= (1-\theta)\,k + (1-k)\,(1-\theta)^{n-l}.
\end{aligned}
$$

In a similar fashion it is easy to show that $\Pr\left(\mathbf{1}_{n-l}|\theta\right) = \theta k + (1-k)\,\theta^{n-l}$. As to the conditional probability of a string of signals containing both ones and zeros, let $\left(\mathbf{0}_{n-l-q}, \mathbf{1}_q, \mathbf{s}_{n-l}\right)$ be an unsorted vector containing $n - l - q$ zeros and q ones where $n > l + q > q > 0$. Then,

$$
\begin{aligned}
\Pr(\mathbf{0}_{n-l-q}, \mathbf{1}_q|\theta) &= \sum_{\mathbf{s}_l \in \{0,1\}^l} \Pr\left(\mathbf{0}_{n-l-q}, \mathbf{1}_q, \mathbf{s}_l|\theta\right), \\
&= (1-\theta)^{n-l-q}\,\theta^q\,(1-k)\sum_{j=0}^{l} \frac{l!}{(l-j)!j!}\,(1-\theta)^j\,\theta^{l-j}, \\
&= (1-k)\,(1-\theta)^{n-l-q}\,\theta^q.
\end{aligned}
$$

∎

Proof of Proposition 3. We want to characterize the incentive condition (A2) for a truth-telling equilibrium under the assumption that the joint probability distribution is given by (9).

The optimal actions taken by the receiver following any string of signals — $y_{\mathbf{1}_l,\mathbf{0}_{n-l}}$, $l = 0, ..., n$ — are given in (11). Let us focus now on the displacement terms commented upon in Section 3.2.1. We begin by noticing that, by definition, it holds

$$\Delta\left(\mathbf{s}_{-i}|s_i\right) = -\Delta\left(\mathbf{s}_{-i}|1 - s_i\right). \tag{A3}$$

For the reader's convenience we report here the values taken by the displacement terms reported in the text, which can be derived using sing (11) and (A3),

$$\Delta\left(\mathbf{0}_{n-1}|0\right) = \frac{6 - k\left(2 - 3n + n^2\right)}{3\left(2 + k\left(n - 1\right)\right)\left(2 + n\right)} = -\Delta\left(\mathbf{1}_{n-1}|1\right), \tag{A4}$$

$$\Delta\left(\mathbf{0}_{n-1-l}, \mathbf{1}_l|0\right) = \frac{1}{2 + n} = -\Delta\left(\mathbf{0}_{n-1-l}\mathbf{1}_l|1\right) \qquad (l \neq 0, n - 1). \tag{A5}$$

For simplicity, let us define the following

$$\Delta_1\left(s_i\right) = \Delta\left(\mathbf{0}_{n-1}|s_i\right) = \Delta\left(\mathbf{1}_{n-1}|s_i\right),$$

$$\Delta_2\left(s_i\right) = \Delta\left(\mathbf{0}_{n-1-l}, \mathbf{1}_l|s_i\right) \qquad (l \neq 0, n - 1).$$

Turning to the probability terms in (8) — which are explicitly visible in (6) —, we notice that, given $\Pr(s_i) = \frac{1}{2}$, the conditional probability terms can be written as

$$\Pr\left(\mathbf{s}_{-i}|s_i\right) = \frac{\Pr\left(\mathbf{s}_{-i}, s_i\right)}{\Pr\left(s_i\right)} = 2\Pr\left(\mathbf{s}_{-i}, s_i\right), \tag{A6}$$

where it is a matter of algebra to show that the unconditional probabilities are

$$\Pr\left(\mathbf{0}_n\right) = \frac{2 + k\left(n - 1\right)}{2\left(n + 1\right)} = \Pr\left(\mathbf{1}_n\right), \tag{A7}$$

$$\Pr\left(\mathbf{0}_{n-l}, \mathbf{1}_l\right) = \frac{\left(1 - k\right)\left(n - l\right)!\, l!}{\left(n + 1\right)!} = \Pr\left(\mathbf{0}_l, \mathbf{1}_{n-l}\right) \qquad (l \neq 0, n - 1). \tag{A8}$$

Notice further that

$$\sum_{l=1}^{n-2}\Pr\left(\mathbf{0}_{n-l-1}, \mathbf{1}_l|s_i\right) = 1 - \Pr\left(\mathbf{0}_{n-1}|s_i\right) - \Pr\left(\mathbf{1}_{n-1}|s_i\right),$$

$$= 1 - 2\Pr\left(\mathbf{0}_{n-1}, s_i\right) - 2\Pr\left(\mathbf{1}_{n-1}, s_i\right),$$

where the last equality follows from (A6). Now define

$$P^* \equiv \Pr\left(\mathbf{0}_{n-1}, s_i\right) + \Pr\left(\mathbf{1}_{n-1}, s_i\right),$$

and notice that it can be easily shown that $P^* = \frac{2(1-k)+kn}{2n}$ for both $s_i = 0, 1$. Summing up, we can rewrite the terms $\mathrm{E}[\Delta\left(\mathbf{s}_{-i}|s_i\right)]$ and $\mathrm{E}[\Delta^2\left(\mathbf{s}_{-i}|s_i\right)]$ in condition (A2) respectively as

$$\sum_{\mathbf{s}_{-i}\in\{0,1\}^{n-1}} \Delta\left(\mathbf{s}_{-i}|s_i\right)\Pr\left(\mathbf{s}_{-i}|s_i\right) = 2P^*\Delta_1\left(s_i\right) + (1-2P^*)\Delta_2\left(s_i\right), \quad\text{(A9)}$$

$$\sum_{\mathbf{s}_{-i}\in\{0,1\}^{n-1}} \Delta^2\left(\mathbf{s}_{-i}|s_i\right)\Pr\left(\mathbf{s}_{-i}|s_i\right) = 2P^*\Delta_1^2\left(s_i\right) + (1-2P^*)\Delta_2^2\left(s_i\right). \quad\text{(A10)}$$

While (A10) is clearly positive, (A9) may be either positive, negative or equal to zero. Moreover, from (A3), whenever (A9) equals to zero for sender i and for signal s_i, it equals to zero for signal $1 - s_i$ and, by the symmetry of (9), for any sender $j \neq i$ and signal s_j.

When the expected displacement (A9) is different from zero, using symmetry and (A4), (A5), (A7) and (A8), straightforward algebra shows that $d^* \equiv d^*\left(n, k\right)$. It can also be easily checked that the denominator of $d^*\left(n, k\right)$ — or, equivalently, the expected displacement (A9) — equals to zero if and only if $k = \bar{k}(n)$. ∎

Proof of Proposition 4. Using the definition of $d^*(n, k)$, some algebra yields

$$\frac{\partial d^*\left(n, k\right)}{\partial k} = \left(n^2 - 1\right)\cdot$$

$$\cdot\frac{k(n-2)\left(k^3(n-2)(n-1)^2(n(n-7)-2)+8k^2(n-1)^2(2(n-4)n-1)+4k(n-1)(13(n-3)n-1)+48(n-4)n\right)-144n}{6(n+2)(k(n-1)+2)^2(k(n-2)(n-1)(k(1+n)-1)-6n)^2},$$

and

$$\lim_{k\to 0}\frac{\partial d^*\left(n, k\right)}{\partial k} = -\frac{n^2 - 1}{6n\left(n + 2\right)} < 0,$$

which proves the first claim of the proposition. Clearly

$$\lim_{n\to\infty}\frac{1}{2\left(1 + n\right)}\left(1 + \sqrt{1 + 24\frac{n(1 + n)}{(n - 2)(n - 1)}}\right) = 0,$$

which proves the second point. And, finally,

$$\lim_{n\to\infty}\left|\frac{36n+k(n-2)(n-1)(12+k(-17+k(n-5)(n-1)(n+1)+n(2n-9)))}{6(2+k(n-1))(2+n)(6n-k(n-2)(n-1)(k(n+1)-1))}\right| = \frac{1}{6}$$

which proves the third result of the proposition. ∎